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DAMPING OF A SEISMICALLY ISOLATED BUILDING BY DRY-FRICTION WEDGEBLOCKS

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The behavior of a seismically isolated building (with rolling support as the isolation element) equipped with tapered dry-friction dampers (wedgeblocks) is described. A specific engineering design is proposed for the dry-friction wedgeblock, where the properties of a heavy "flat" damper are preserved in dampers of comparatively small mass.

The objective of this paper is to describe the behavior of a seismically isolated building (with seismic isolation provided by a rolling support) equipped with tapered dry-friction dampers (wedgeblocks).

The behavior of a seismically isolated building is described by a model discussed in a paper [1] treating small vibrations of the system "building–support–foundation." The frequency of the vibrations of the system is determined by the geometrical parameters of the rolling supports adopted as seismic isolation elements. When driven externally (seismically) at the same frequency as its normal mode, the system enters into resonance, which requires the application of auxiliary technological or structural devices to eliminate it. The so-called flat[*] dry-friction dampers (shimblocks) used in the majority of structural solutions are ineffective, because the dampers must be heavy, and an increase in mass conflicts with the whole idea of seismic isolation of buildings.

In this article we propose a specific engineering design for a tapered dry-friction damper, or wedgeblock, which has a comparatively low mass and yet preserves the properties of a heavy "flat" damper.

1. Damper with Variable Dry Friction. A damper with variable dry friction comprises a body having a certain mass m, which rests on an inclined plane with a specified sliding friction coefficient and a specified [2] (or variable [3]) angle of inclination α and is capable of moving upward along the inclined plane under the influence of a horizontally directed force P and of sliding downward under the influence of its own weight (Figs. 1 and 2).

We consider the special case of a damper with variable dry friction, i.e., a dry-friction wedgeblock. When a horizontal external force P acts on the upwardly moving wedgeblock, the sliding friction force becomes dependent on the magnitude of this external force:

$$F_{\text{ff}} = fN = f\left(mg\cos\alpha + P\sin\alpha\right).$$

When the mass moves downward, the friction force is given by the same expression, but is directed upward. We have

$$\vec{F}_{\text{ff}} = -fN\frac{\vec{V}}{\left|\vec{V}\right|},$$

where $\vec{V}$ is the relative velocity of the moving mass. In this case, the normal reaction of the sliding surface depends on the external force P.

[*] A "flat" damper is interpreted as one that is driven in a horizontal plane by a sliding friction force of constant absolute value (Coulomb friction law).

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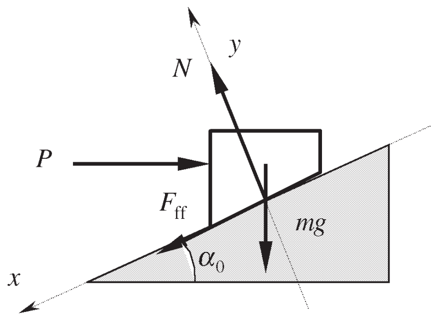

Fig. 1

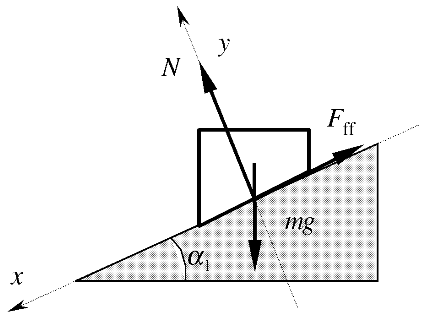

Fig. 2

Thus, the wedgeblock must move upward along the inclined sliding surface under the influence of a certain horizontally directed force and slide downward along the same surface without this force under the influence of its own weight. To meet these conditions, it is necessary to choose the right angle of inclination of the sliding surface, based on the condition of limiting equilibrium of the wedgeblock sitting on the inclined sliding surface. We find the limiting angle, above which it is impossible for the wedgeblock to move upward under the influence of any horizontally directed force.

Writing the equilibrium equations and expressing the force *P*, we obtain

$$P = \frac{mg(f\cos\alpha + \sin\alpha)}{(\cos\alpha - f\sin\alpha)}.$$

When the denominator is equal to zero, the force *P* becomes infinitely large, i.e., the wedgeblock never shifts its position when the angle of inclination exceeds the limit:

$$\alpha_0 \geq \text{arccot}(f).$$

The limiting angle changes for different sliding surfaces (see Table 1).

For the wedgeblock to return to its original position after cessation of the external force, it is necessary to determine the angle at which the wedgeblock sitting on the inclined sliding surface will slide downward under the influence of its own weight.

Inasmuch as the motion is confined strictly along the *x* axis, the condition for motion of the wedgeblock under the influence of its own weight has the form

$$mg\sin\alpha_1 > fmg\cos\alpha_1,$$

and the minimum angle α_1 is given by the expression

$$\alpha_1 \geq \arctan(f).$$

TABLE 1

Friction coefficient f	Limiting angle (deg)	Minimum sliding angle (deg)
≈0.1 (Teflon–metal)	84.3	5.71
≈0.25 (metal–metal	75.9	14.0
≈0.5 (concrete–metal)	63.4	26.6
≈0.7 (concrete–concrete)	55.0	34.0

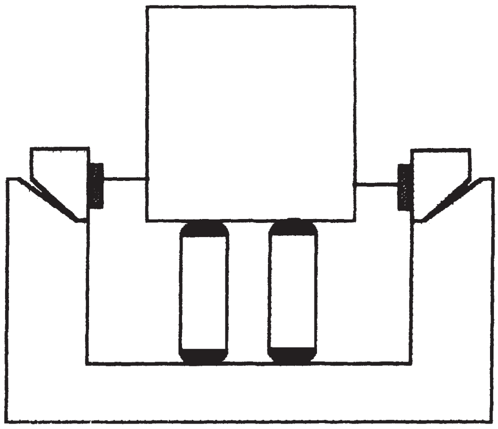

Fig. 3. Computational scheme.

Table 1 shows the limiting and minimum sliding angles for various friction coefficients of materials most commonly used in construction.

Consequently, the calculated angle of inclination of the sliding surface must fall between the limits $\alpha_1 < \alpha < \alpha_0$ for the case described above.

2. Kinematic Foundation with a Dry-Friction Wedgeblock. We consider a seismic isolation support unit for an earthquake-resistant building; it consists of a supporting kinematic foundation [1] and dry-friction wedgeblocks [2] (see Fig. 3).

The computational scheme has the following specific characteristics [1]:

1) the stiffness of the seismic isolation system is determined by the force of its own weight as in a mathematical pendulum;

2) the building is regarded as a perfectly rigid solid;

3) the stiffness of the support elements is commensurate with the stiffness of the structures situated above them;

4) the dissipative force is Coulomb friction.

The description of each design solution of the seismically isolated building requires the specific formulation of an equation of motion, because the design solution imposes definite conditions on the formulation of the equation of motion.

For the proposed computational scheme, the equation of motion has the form

$$M\ddot{x} + \left(Mg\left(1 + \frac{W_Z}{g}\right) - \operatorname{sgn} \dot{Z}\Phi_{\text{ff}} \right)\frac{h}{H^2}x + P_X = -W_X M, \tag{1}$$

where $\Phi_{\text{ff}} = f_{\text{bg}}P_X$,

$$P_X = \frac{\ddot{x}m(1 + \tan\alpha C \operatorname{sgn} x) + mg\left(1 + \frac{W_Z}{g}\right)C + W_X m}{1 - f_{\text{bg}}C \operatorname{sgn} \dot{x} \operatorname{sgn} x}, \tag{2}$$

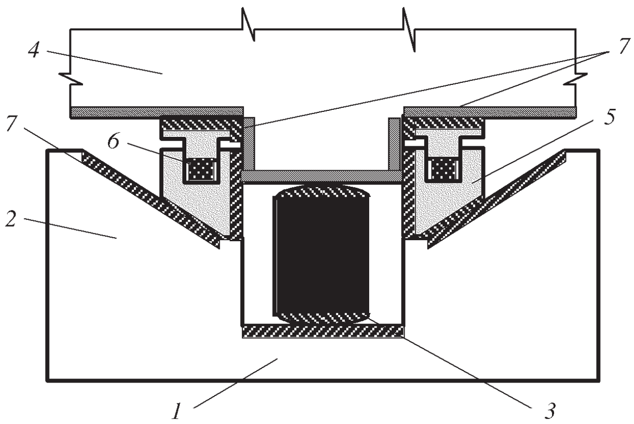

Fig. 4. Seismic isolation support unit for a building: *1*) lower support block; *2*) can; *3*) seismic isolation post with spherical ends; *4*) upper support block; *5*) wedgeblock damper; *6*) elastic element; *7*) fittings.

$$C = \frac{f_{\mathrm{wb}} \operatorname{sgn} \dot{x} + \tan\alpha \operatorname{sgn} x}{1 - f_{\mathrm{wb}} \tan\alpha \operatorname{sgn} \dot{x} \operatorname{sgn} x},$$

M is the mass of the seismically isolated building, x is the relative displacement of the building, g is the free-fall acceleration, W_X and W_Z are the horizontal and vertical components of the acceleration of the foundation, respectively, f_{bg} is the sliding friction coefficient along the vertical surface of the wedgeblock, f_{wb} is the sliding friction coefficient of the wedgeblock along the inclined surface, $\dot{Z}$ is the vertical velocity of the wedgeblock along the oblique guide track relative to the elevation of the seismically isolated building, P_X is the force exerted on the building by the activated wedgeblocks, and h and H are parameters that depend on the type of seismic isolation system (in our case of rolling supports h is the thickness of the ejected layer, and H is the height of the post).

For the given design solution, we have $\dot{Z} = \dot{x}\tan\alpha\operatorname{sgn}x$, so that $f_{\mathrm{bg}}\operatorname{sgn}\dot{Z} = f_{\mathrm{bg}}\operatorname{sgn}\dot{x}\operatorname{sgn}x$; the parameters P_X and $\dot{Z}$ can differ for other design solutions.

In contrast with the equation of motion of a seismically isolated building equipped with a "flat" dry-friction damper [1]:

• Equation (1) contains the force P_X instead of a term expressing the dry friction;

• without an external driving force the dissipative force P_X has a constant absolute value, and the free vibrations of the given system do not differ from the free vibrations using "flat" dry-friction dampers except that, given damping elements of the same mass and identical sliding friction coefficients, the vibrations decay far more rapidly (owing to the angle of inclination of the sliding surface, whose variation is analogous to an increase in the mass of the dampers in the conventional situation);

• in the presence of an external driving force, the dissipative force P_X is no longer constant in absolute value, but varies as a function of the externally (seismically) induced accelerations W_X and W_Z (2).

We now give the limiting and minimum angles of inclination of the sliding surface for the design solution [2] (Fig. 4) described by the vibration equation (1). It is evident from Fig. 4 that the interaction of the dry-friction wedgeblock with the building produces another element: a vertical sliding surface with the friction coefficient f_{bg}. Consequently, the slope parameters of the sliding surface in this case, in contrast with the isolated wedgeblock discussed above, lead to the expression

$$\arctan\left(f_{\mathrm{wb}}\right) < \alpha < \arctan\left(\frac{1 - f_{\mathrm{bg}} f_{\mathrm{wb}}}{f_{\mathrm{bg}} + f_{\mathrm{wb}}}\right).$$

3. Calculation of the Free Vibrations of a Building on Rolling Supports with Dry-Friction Wedgeblocks. We have calculated the free vibrations of a seismically isolated building with dry-friction wedgeblocks having the following parameters: $H = 0.5$ m, $h = 0.1$ m for a damper-to-building mass ratio $m/M = 0.001$, a sliding friction coefficient $f = 0.1$, an initial velocity $V_0 = 0.1$ m/s, and two angles of inclination of the sliding surface $\alpha = 50°$ and $\alpha = 70°$. This case corresponds to Eq. (1) with a

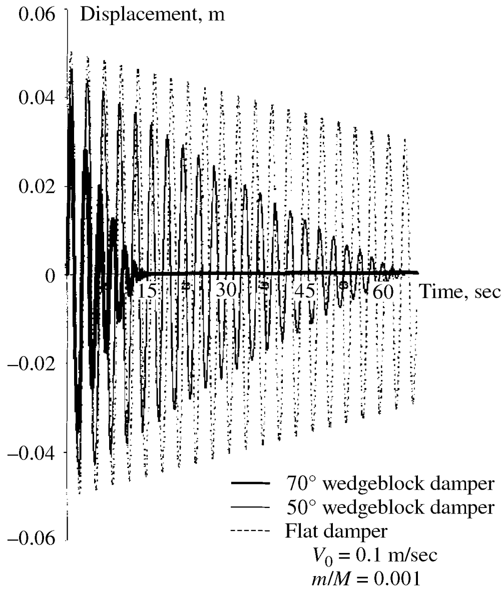

Fig. 5

zero right-hand side, which has been solved by the Runge–Kutta method to fourth-order accuracy. We have also calculated the free vibrations of a structure using a "flat" dry-friction damper with the same parameters.

The solid curves in Fig. 5 represent the results of the calculations of the free vibrations of a seismically isolated building with wedgeblocks for sliding angles $\alpha = 50°$ and $\alpha = 70°$, and for comparison the dashed curves represent the results for damped vibrations of the same structure with a conventional flat damper.

It is evident from the figure that the use of wedgeblocks greatly increases the vibration decay rate. For example, the vibrations die out in 15 s for $\alpha = 70°$, whereas for the flat damper the amplitude of the vibrations changes insignificantly in the same time interval. It is also evident that the vibration decay rate depends on the wedge angle of the damper: the vibrations take almost seven times as long to die out for $\alpha = 50°$ than for $\alpha = 70°$.

The calculations reveal an important advantage of the wedgeblock over the conventional flat damper: a substantial decrease in the mass of the damper. We note that the mass of the flat damper would have to be 100 times the mass of the wedgeblock with $\alpha = 70°$ to extinguish a building equipped with the latter.

We can make a simple estimate in this regard. If we assume that the mass of a nine-story building is approximately 3000 tonnes, the ratio of the mass of the flat damper to that of the building is equal to 0.1, and the corresponding ratio for the wedge block is equal to 0.001, we find that the masses of the flat damper and the wedgeblock are equal to 300 tonnes and 3 tonnes, respectively. The savings in construction material are obvious.

An analysis of the results for normal-mode vibrations supports the idea of using a wedgeblock, whereby the extinction of vibrations can be enhanced by varying the angle of inclination of the sliding surface with dry-friction dampers having a low mass.

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